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The ^{15}N correction to the measured asymmetries

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1 Introduction

The presence of ^{15}N in the ammonia used in the polarized targets introduces an unwanted asymmetry because the nitrogen is partly polarized relative to the hydrogen or deuterium. To correct the proton or deuteron asymmetry we need to estimate or measure the nitrogen contribution to the measured asymmetry. The corrections for each target type are discussed below

2 Proton

For the proton measurements on ordinary ammonia NH_3, the ^{15}N contribution can be determined from our measured asymmetry in the region below the $e-p$ elastic peak where no proton asymmetry is expected because scattering on free protons is forbidden.

We call $A^M(W)$ the measured counts asymmetry ε corrected for the beam P_b and (proton) target P_1 polarizations

$$A^M(W) = \frac{\varepsilon(W)}{P_b P_1} \tag{1}$$

$A^M(W)$ for protons is tabulated for $W < 1.073$ GeV in the output files of the analysis. $A^M(W < W_{el})$ depends only on the nitrogen contribution, where W_{el} is the lowest value of invariant mass for elastic scattering. $W_{el} = 0.85$ GeV is a value that works for both parallel and perpendicular asymmetries.

The expected number of counts from all the target components for each beam helicity $L(W)$ or $R(W)$ for every bin in W can be written as

$$L(R) = \Phi(N_{15}\sigma_{15}^{L(R)} + N_1\sigma_1^{L(R)} + \sum N_A\sigma_A) \tag{2}$$

where Φ is a flux factor, N_A are the numbers of scattering nuclei of mass A, $\sigma_A^{L(R)}(W)$ represent polarized $e - nucleus$ cross sections (elastic or inelastic for H, quasielastic or inelastic for N). All unpolarized nuclei are lumped in the sum. The polarized cross sections can be written in terms of nucleon polarized and unpolarized cross sections

$$\sigma_A^{L(R)}(W) = \quad (Z - Z^{pol})\sigma_{p(A)} + (N - N^{pol})\sigma_{n(A)} + Z^{pol}\sigma_{p(A)}^{L(R)} + N^{pol}\sigma_{n(A)}^{L(R)} \tag{3}$$

where Z, N refer to the total numbers of protons and neutrons in the nucleus and Z^{pol}, N^{pol} represent the polarized ones; $\sigma_{p,n(A)}$ are $e{-}nucleon$ unpolarized cross sections for nuclear species A. The polarized nucleon cross sections $\sigma_{p,n(A)}^{L(R)}$ can be expressed in terms of unpolarized nucleon cross sections $\sigma_{p,n(A)}$ and nucleon asymmetries $A_{p,n(A)}(W)$

$$\sigma_{p,n(A)}^{L(R)}(W) = \sigma_{p,n(A)}(A)(1 \pm P_b P_A A_{p,n(A)}(W)) \tag{4}$$

where P_b is the beam polarization and P_A is the polarization of nucleus A.

For ^{15}N which has only one polarizable proton, we can simplify this notation using $\sigma_{15}^{L(R)} = \sigma_{15} \pm \sigma_{p(15)} P_b P_{15} A_{15}$, where $A_{15} \equiv A_{p(15)}$. The rates for each helicity are

$$L(R) = \Phi(N_{15}(\sigma_{15} \pm \sigma_{p(15)} P_b P_{15} A_{15}) + N_1\sigma_1(1 \pm P_b P_1 A_p) + \sum N_A \sigma_A), \tag{5}$$

where P_1, P_{15} are the hydrogen and nitrogen polarizations.

The difference over the sum of counts for each helicity is the counts asymmetry $\varepsilon(W)$

$$\varepsilon = \frac{L - R}{L + R} = \frac{P_b(N_{15}\sigma_{p(15)} A_{15} P_{15} + N_1 \sigma_1 A_p P_1)}{N_{15}\sigma_{15} + N_1\sigma_1 + \sum N_A \sigma_A} \tag{6}$$

The customary approach is to factor out the product $N_1\sigma_1$ and write ε in terms of the proton dilution factor f_1

$$\begin{aligned} \varepsilon &= f_1 P_b(\frac{N_{15}\sigma_{p(15)}}{N_1\sigma_1} A_{15} P_{15} + A_p P_1) \\ f_1 &= \frac{N_1\sigma_1}{N_{15}\sigma_{15} + N_1\sigma_1 + \sum N_A \sigma_A} \end{aligned} \tag{7}$$

However, this form becomes undefined for $\sigma_1(W < W_{el}) = 0$. The alternative solution is not to factor out $N_1\sigma_1$ but define a nitrogen dilution factor f_{15}, as used by C. Harris [1]

$$\begin{aligned} \varepsilon(W) &= P_b(f_{15}(W)P_{15}A_{15}(W) + f_1(W)P_1A_p(W)); \\ f_i(W) &= \frac{N_i\sigma_i(W)}{N_{15}\sigma_{15}(W) + N_1\sigma_1(W) + \sum N_A\sigma_A(W)} \end{aligned} \tag{8}$$

where i= 1 or $p(15)$.

With (8) it becomes possible to correct the measured asymmetry for the nitrogen contribution, using the average nitrogen asymmetry $\langle A_{15}(W)\rangle$ measured for $W < W_{el}$, assuming that the asymmetry is independent of W:

$$\begin{aligned} \varepsilon(W < W_{el}) &= P_bP_{15}f_{15}(W < W_{el})A_{15}(W < W_{el}) \\ &= P_bP_1A^M(W < W_{el}) \end{aligned} \tag{9}$$

in terms of the measured asymmetry defined in (1). The nitrogen asymmetry is then

$$\langle A_{15}(W < W_{el})\rangle = \frac{P_1}{P_{15}}\langle\frac{A^M(W < W_{el})}{f_{15}(W < W_{el})}\rangle \tag{10}$$

and the proton asymmetry for $W \geq W_{el}$ can be obtained from

$$\varepsilon(W) = P_b(f_{15}(W)P_{15}\langle A_{15}\rangle + f_1(W)P_1A_p(W)) \tag{11}$$

or

$$\begin{aligned} A_p(W) = & \quad \frac{1}{f_1(W)}\Big(\frac{\varepsilon(W)}{P_bP_1} - f_{15}(W)\frac{P_{15}}{P_1}\langle A_{15}\rangle\Big) \\ = & \quad \frac{1}{f_1(W)}\Big(A^M(W) - f_{15}(W)\langle\frac{A^M(W < W_{el})}{f_{15}(W < W_{el})}\rangle\Big). \end{aligned} \tag{12}$$

A_{15} can also be estimated from models of ^{15}N [2]. From the angular momentum decomposition of the $p_{1/2}$ level that is populated by the unpaired proton in the single particle shell model, one expects $A_{15}^{model}(W) = P_{p(15)}A_p(W) = -A_p(W)/3$, where A_p is the proton asymmetry and the factor $-1/3$ is the effective polarization of the unpaired proton in nitrogen $P_{p(15)}$. In order to solve for A_p using A_{15}^{model} one needs to go back to (8),

$$\begin{aligned}\varepsilon(W) = & \quad P_b(-\frac{1}{3}f_{15}(W)P_{15}A_1(W) + f_1(W)P_1A_p(W)) \\ = & \quad P_bP_1f_1(W)(-\frac{1}{3}\frac{f_{15}(W)}{f_1(W)}\frac{P_{15}}{P_1} + 1)A_p(W) \\ = & \quad P_bP_1f_1(W)C_NA_p(W)\end{aligned} \tag{13}$$

and the customary nitrogen correction C_N for DIS is recovered

$$\begin{aligned}C_N &= 1 - \frac{1}{3}\frac{f_{15}(W)}{f_1(W)}\frac{P_{15}}{P_1} \\ &= 1 - \frac{1}{3}\frac{1}{3}\frac{P_{15}}{P_1}g_{EMC}(W)\end{aligned} \tag{14}$$

where the last form comes from

$$\frac{f_{15}(W)}{f_1(W)} = \frac{N_{15}}{N_1}\frac{\sigma_{p(15)}}{\sigma_1} = \frac{1}{3}g_{EMC}(W) \tag{15}$$

where the ratio of the DIS $e-p$ cross sections on nitrogen and on hydrogen is approximated by the EMC effect parameterization $g_{EMC}(W)$ (more correctly $g_{EMC}(x)$).

It is important to keep track of the opposite relative signs of P_{15} and P_1. P_1 and P_{15} are related by fits to experimental data, for example

$$|P_{15}|\% = -\Big(0.0312\% + 5.831\times10^{-2}|P_1| + 8.935\times10^{-4}|P_1|^2 + 8.685\times10^{6}|P_1|^3\Big) \tag{16}$$

which is based on E143 plus PSI measurements of positive and negative enhancements. P_1 needs to be in %.

Applying this correction to the asymmetries in the resonances, where

$$A^M(W) = \frac{\varepsilon(W)}{f_1(W)P_tP_b} \tag{17}$$

the inelastic proton asymmetry is

$$A_p(W) = \frac{A^M(W)}{C_N}. \tag{18}$$

This form actually is valid at any $W \geq W_{el}$, with

$$C_N = 1 + \frac{A_{15}(W)}{A_p(W)}\frac{f_{15}(W)}{f_1(W)}\frac{P_{15}}{P_1}. \tag{19}$$

If the expected value of A_p is known from other measurements, such as the elastic asymmetry $A_p = A^{el}(e-p)$, the absolute size of the nitrogen asymmetry can be estimated from A_{15}^{model} to compare with the measured quantity

$$A^M(W) = \frac{\varepsilon(W)}{P_b P_1} = f_1(W)A_p(W) - \frac{1}{3}f_{15}(W)\frac{P_{15}}{P_1}A_p(W) \tag{20}$$

For example, for $W < W_{el}$ where only nitrogen contributes ($f_1 = 0$), $f_{15} \simeq 0.7$, $P_{15}/P_1 \sim -0.165$, $A_p = 0.21$ for parallel data, and $A^M(W < W_{el}) \simeq 0.008$. For perpendicular data, $A_p = -0.103$ and $A^M(W < W_{el}) \simeq -.004$

The predicted values disagree with the measured values $A^M(W < W_{el}) \simeq 0.0029 \pm 0.0024$ for parallel data and $A^M(W < W_{el}) \simeq 0.010 \pm 0.003$ for perpendicular data. This is an indication that the simple model prediction may be insufficient.

2.1 ^{15}N model extension

The model can be readily extended to include mixing of the $p_{3/2}$ state (only the $M = \pm 1/2$ substates are allowed, to preserve the total nuclear ^{15}N spin $I = (1/2)^-$). The $M = \pm 1/2$ angular momentum substates are (notation C-G$|m_l, m_s; M\rangle$)

$$\begin{aligned} \sqrt{\frac{1}{3}}|1,-1/2;\ 1/2\rangle &+ \sqrt{\frac{2}{3}}|0,\ 1/2;\ \ 1/2\rangle \\ \sqrt{\frac{1}{3}}|-1,1/2;-1/2\rangle &+ \sqrt{\frac{2}{3}}|0,-1/2;-1/2\rangle \end{aligned} \tag{21}$$

It is easily seen that 2/3 of the time the proton and N have parallel spins ($m_s = +M$,) and 1/3 anti parallel, for a net 1/3 of the time having parallel spins in the $M = \pm 1/2$ substates.

The net normalized number of aligned protons in ^{15}N can be written in terms of the contributions of the two states $p_{1/2}$ and $p_{3/2}$

$$\frac{N_\parallel}{N} = -\frac{1}{3}P_{p(1/2)} + \frac{1}{2}\frac{1}{3}P_{p(3/2)} \tag{22}$$

where P_i represents the probability of the proton being in each state ($\sum P_i = 1$,) and the extra 1/2 reflects the restriction to $M = \pm 1/2$ substates.

Using (10) and substituting the measured values, one gets $A_{15} = -0.026$ for parallel data and $A_{15} = 0.086$ for perpendicular data. Since the substate probabilities P_i add to unity, there is only one unknown,

$$\frac{N_\parallel}{N} = -\frac{1}{3} + \frac{1}{2}P_{p(3/2)} = \frac{A_{15}}{A_p} \tag{23}$$

where A_p is the elastic asymmetry for free protons. $P_{3/2} = 0.42$ for parallel data but it is negative (-0.91) for perpendicular, indicating that the ground state of nitrogen may be more complicated than the model.

3 Deuteron

For ND_3, we start with the counts asymmetry, including the contributions of ^{15}N, ^{14}N and unsubstituted protons, all of which polarize together with the deuterium

$$\varepsilon = \frac{L-R}{L+R} = \frac{P_b(N_2P_2\sigma_2A_d + N_{15}P_{15}\sigma_p^{15}A_p^{15} + N_{14}P_{14}(\sigma_p^{14}A_p^{14} + \sigma_n^{14}A_n^{14}) + N_1P_1\sigma_1A_1)}{N_{15}\sigma_{15} + N_2\sigma_2 + \sum N_A\sigma_A}, \tag{24}$$

where the notation has been changed slightly, $\sigma_p^A \equiv \sigma_{p(A)}$, etc., to fit in the margins. Collecting the common terms, we have

$$\varepsilon = f_2P_b(P_2A_d + \frac{N_{15}\sigma_{p(15)}}{N_2\sigma_2}P_{15}A_p^{15} + \frac{N_{14}}{N_2\sigma_2}P_{14}(\sigma_p^{14}A_p^{14} + \sigma_n^{14}A_n^{14}) + \frac{N_1\sigma_1}{N_2\sigma_2}P_1A_1) \tag{25}$$

$$f_2 = \frac{N_2\sigma_2}{N_{15}\sigma_{15} + N_2\sigma_2 + \sum N_A\sigma_A}.$$

We need to write $A_{p,n}^{14,15}$ in terms of proton A_p and neutron A_n asymmetries. For A_p^{15} we use the SPS model discussed earlier for NH_3 $A_{15}(W) = -A_1(W)/3$. For $A_{p,n}14$ we have contributions from the proton and the neutron. However, there are no data on A_n at RSS kinematics, so we make use of the relation between the deuteron asymmetry and the proton and neutron asymmetries $\sigma_2 A_2 = \gamma_2(\sigma_1 A_1 + \sigma_n A_n)$ to solve for A_n

$$\sigma_p^{14} A_p^{14} + \sigma_n^{14} A_n^{14} = -\frac{1}{3}\Big(\sigma_{p(14)} A_p (1 - \frac{\sigma_{n(14)}}{\sigma_{n(2)}} \frac{\sigma_{p(2)}}{\sigma_{p(14)}}) + \frac{\sigma_2 A_d}{\gamma_2} \frac{\sigma_{n(14)}}{\sigma_{n(2)}}\Big),$$

The factor $\gamma_2 = 0.924$ represents the effective polarization of the nucleons in the deuterons (not exactly unity due to the deuteron D-state, where the nucleons are aligned antiparallel with respect to the deuteron spin [2]).

The ratio $(\sigma_{n(14)}\sigma_{p(2)})/(\sigma_{n(2)}\sigma_{p(14)})$ is almost exactly one, so we neglect the term involving A_p. Substituting and collecting terms

$$\begin{aligned} \varepsilon = & \quad f_2 P_b (P_2 A_d - \frac{1}{3} \frac{N_{15}\sigma_p^{15}}{N_2 \sigma_2} P_{15} A_1 - \frac{1}{3} \frac{N_{14}}{N_2} \frac{1}{\gamma_2} \frac{\sigma_n^{14}}{\sigma_n^{(2)}} P_{14} A_d + \frac{N_1 \sigma_1}{N_2 \sigma_2} P_1 A_1) \\ = & \quad f_2 P_b P_2 \Big((1 - \frac{1}{3} \frac{N_{14}}{N_2} \frac{1}{\gamma_2} \frac{P_{14}}{P_2} \frac{\sigma_n^{14}}{\sigma_n^{(2)}}) A_d - (\frac{1}{3} \frac{N_{15}\sigma_p^{15}}{N_2 \sigma_2} \frac{P_{15}}{P_2} - \frac{N_1 \sigma_1}{N_2 \sigma_2} \frac{P_1}{P_2}) A_1)\Big) \end{aligned} \tag{26}$$

Before proceeding to the full solution of eq.(26) for A_d, we can estimate the magnitude of the corrections involved, by looking at the numerical values of the different factors as they apply to RSS.

Table 1. Numeric values of factors.

Labels	Values	Ratio	Alternative ratio*
N_1/N_2	1%/99%	0.01	
N_{14}/N_2	2%/(3*99%)	0.0067	
N_{15}/N_2	98%/(3*99%)	.330	
P_1/P_2		4.4	2
P_{14}/P_2		0.48	0.33
P_{15}/P_2		-0.50	-0.4

*Polarization ratio from EST; alternative ratio from E143 technical run.

The main correction comes from the ^{15}N contribution. Neglecting the others one gets

$$
\begin{aligned}
\varepsilon = &\quad f_2 P_b P_2 \Big(A_d - \frac{1}{3} \frac{N_{15}\sigma_p^{15}}{N_2\sigma_2} \frac{P_{15}}{P_2} A_1 \Big) = f_2 P_b P_2 \Big(A_d - \frac{1}{3} 0.33 \times (-0.5) \frac{\sigma_p^{15}}{\sigma_2} A_p \Big) \\
= &\quad f_2(W) P_b P_2 A_d(W) \Big(1 + 0.055 \frac{\sigma_p^{15}(W)}{\sigma_2(W)} \frac{A_p(W)}{A_d(W)} \Big) \\
\simeq &\quad f_2(W) P_b P_2 A_d(W)(1.055) \qquad (27)
\end{aligned}
$$

taking the proton to deuteron resonances cross section ratio ~ 0.5, and the RSS measured ratio of raw proton to deuteron asymmetries ~ 2. So the ^{15}N correction represents a $\sim 6\%$ relative reduction to the measured deuteron asymmetry.

Collecting the coefficients of A_1 and A_d into

$$
\begin{aligned}
C_1(W) &= \frac{1}{3} \frac{N_{15}\sigma_p^{15}(W)}{N_2\sigma_2(W)} \frac{P_{15}}{P_2} - \frac{N_1\sigma_1(W)}{N_2\sigma_2(W)} \frac{P_1}{P_2} \\
C_d(W) &= 1 - \frac{1}{3} \frac{N_{14}}{N_2} \frac{1}{\gamma_2} \frac{P_{14}}{P_2} \frac{\sigma_n^{14}(W)}{\sigma_n^{(2)}(W)} \qquad (28)
\end{aligned}
$$

the deuteron asymmetry corrected for the contributions of other polarized nuclei is

$$
A_d(W) = \frac{1}{C_d(W)} \Big(\frac{\varepsilon(W)}{f_2(W) P_b P_2} + C_1(W) A_1(W) \Big) \qquad (29)
$$

Ignoring for the time being the W dependence of $C_{1,2}$, we can estimate the relative size of these corrections:

$$
\begin{aligned}
C_1 &\sim \frac{1}{3} \times 0.330 \times 0.5 \times (-0.5) - 0.01 \times 0.5 \times 4.4 = -0.050 \\
C_d &= 1 - \frac{1}{3} \times 0.007 \frac{1}{0.924} 0.48 \times 1 = 1 - 0.001 \qquad (30)
\end{aligned}
$$

from which we conclude that the most important correction is C_1; C_d is entirely negligible.

3.1 Quasi-elastic (QE) region

In the QE region the contribution of the unsubstituted protons is not as negligible as in the inelastic, because the cross section σ_1 is the elastic peak but

σ_p^{15} is the e–^{15}N quasielastic one. Also, the asymmetry A_1 is the elastic one but for the bound ^{15}N proton it is a quasielastic asymmetry. This requires to split $C_1(W)$ into two separate coefficients

$$C_{15}(W) = \frac{1}{3}\frac{N_{15}\sigma_p^{15}(W)}{N_2\sigma_2(W)}\frac{P_{15}}{P_2}$$
$$C_1(W) = \frac{N_1\sigma_1(W)}{N_2\sigma_2(W)}\frac{P_1}{P_2} \quad (31)$$

so the deuteron quasielastic asymmetry corrected for the contributions of other polarized nuclei is

$$A_d(W) = \frac{1}{C_d(W)}\Big(\frac{\varepsilon(W)}{f_2(W)P_bP_2} + C_{15}(W)A_1(W) - C_1(W)A_p(W)\Big) \quad (32)$$

Here $A_1(W)$ is the proton QE asymmetry and A_p is the elastic asymmetry.

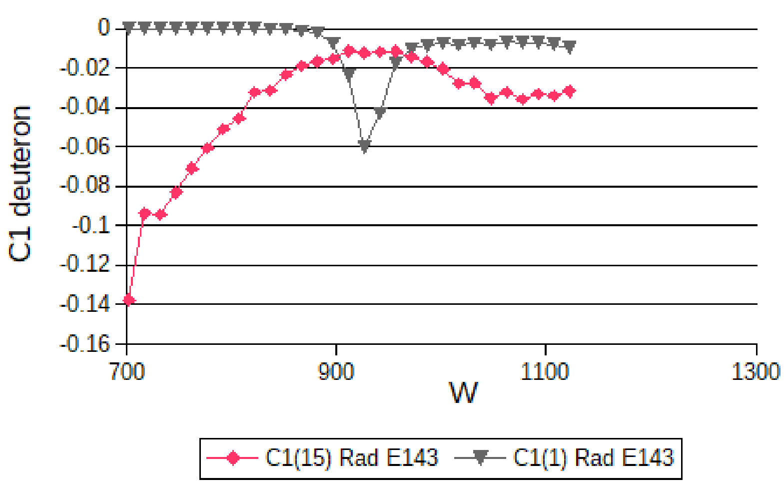

Figure 1: $C_{15}(W)$ and $C_1(W)$ plotted vs W in the QE region. Cross sections from radiated MC rates. Numbers of centers and polarizations from table 1.

References

[1] C. Harris, Thesis, unpublished.

[2] O. Rondon, Phys. Rev. C **60**, 035201 (1999).