

TMD EXTRACTION WITH GLOBAL FITS & ANN

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OUTLINE

- A brief Introduction to TMD PDFs
- Sivers Function
- Sivers asymmetry from SIDIS
- Sivers asymmetry from DY
- Global analyses of Sivers function
- Fitting methodology
- Neural Network approach with SIDIS
- Fit results to SIDIS
- Fit results to SIDIS & DY
- Discussion & Future work

TMD PDFs

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1 = \odot$	N/A	$h_1^\perp = \odot - \ominus$ Boer-Mulders
	L	N/A	$g_{1L} = \odot - \ominus$ Helicity	$h_{1L}^\perp = \odot - \ominus$
	T	$f_{1T}^\perp = \odot - \ominus$ Sivers	$g_{1T}^\perp = \odot - \ominus$	$h_1 = \odot - \ominus$ $h_{1T}^\perp = \odot - \ominus$ Transversity

$$\Phi(x, k_T; S) = \int \frac{d\xi^- d\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S | \bar{\psi}(0) \mathcal{U}_{[0, \xi]} \psi(\xi) | P, S \rangle |_{\xi^+ = 0}$$

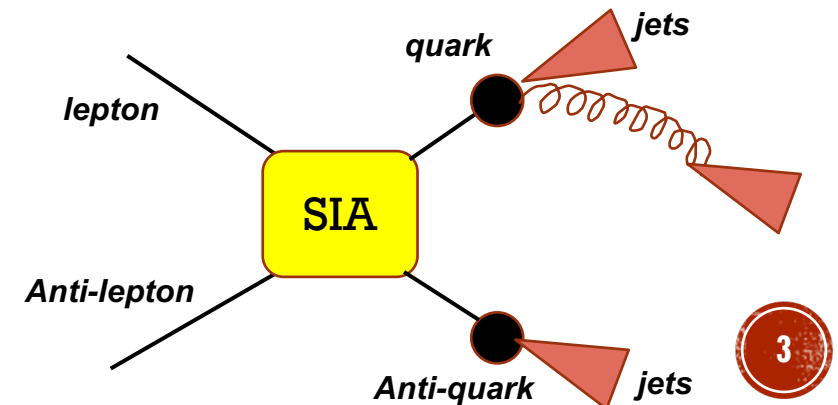
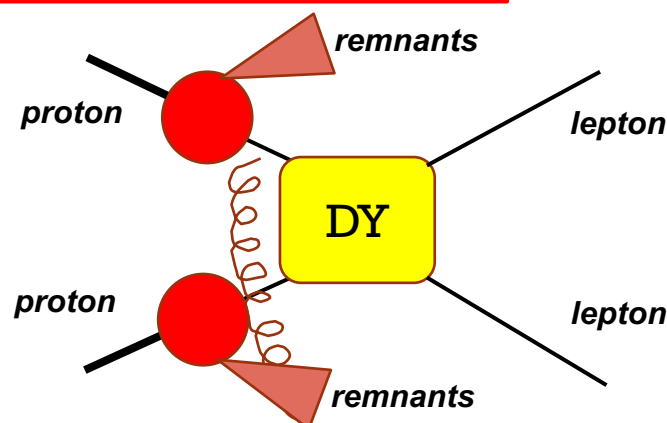
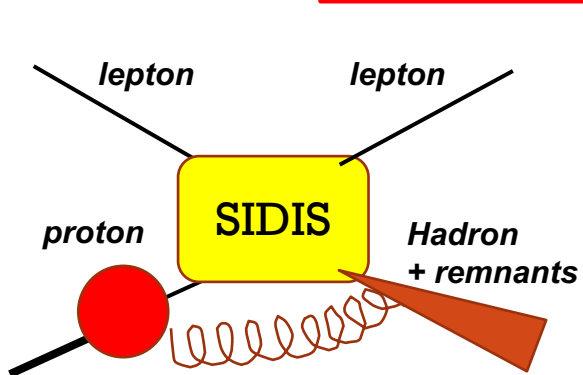
Quark correlator can be decomposed into 8 components
(6 T-even and 2 T-odd terms) at leading-twist

$$\begin{aligned} \Phi(x, k_T, P, S) = & f_1(x, k_T^2) \frac{\not{P}}{2} + \frac{h_{1T}(x, k_T^2)}{4} \gamma_5 [\not{S}_T, \not{P}] + \frac{S_L}{2} g_{1L}(x, k_T^2) \gamma_5 \not{P} + \frac{k_T \cdot S_T}{2M} g_{1T}(x, k_T^2) \gamma_5 \not{P} \\ & + S_L h_{1L}^\perp(x, k_T^2) \gamma_5 \frac{[k_T, \not{P}]}{4M} + \frac{k_T \cdot S_T}{2M} h_{1T}^\perp(x, k_T^2) \gamma_5 \frac{[k_T, \not{P}]}{4M} \end{aligned}$$

T-even

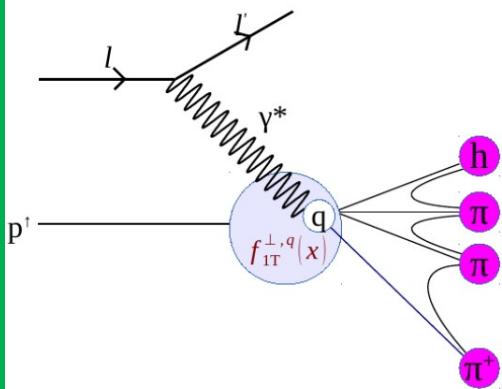
$$+ i h_1^\perp(x, k_T^2) \frac{[k_T, \not{P}]}{4M} - \frac{\epsilon_T^{k_T S_T}}{4M} f_{1T}^\perp(x, k_T^2) \not{P}$$

T-odd



TMD PDFS

Polarized Semi-Inclusive DIS



SIDIS

$$\frac{d\sigma_{SIDIS}^{LO}}{dx dy dz dp_T^2 d\varphi_h d\psi} = \left[\frac{\alpha}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \right] \times (F_{UU,T} + \varepsilon F_{UU,L}) \left\{ 1 + \cos 2\phi_h (\varepsilon A_{UU}^{\cos 2\phi_h}) \right. \\ \left. + S_T \begin{bmatrix} \sin(\phi_h - \phi_S) (A_{UT}^{\sin(\phi_h - \phi_S)}) \\ + \sin(\phi_h + \phi_S) (\varepsilon A_{UT}^{\sin(\phi_h + \phi_S)}) \\ + \sin(3\phi_h - \phi_S) (\varepsilon A_{UT}^{\sin(3\phi_h - \phi_S)}) \end{bmatrix} \right\}$$

PDF \otimes FF

$$A_{UU}^{\cos 2\phi_h} \propto h_1^{\perp q} \otimes H_{1q}^{\perp h}$$

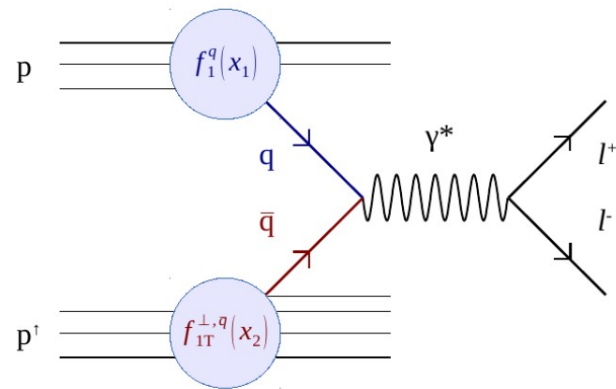
$$A_{UT}^{\sin(\phi_h - \phi_S)} \propto f_{1T}^{\perp q} \otimes D_{1q}^h$$

$$A_{UT}^{\sin(\phi_h + \phi_S)} \propto h_1^q \otimes H_{1q}^{\perp h}$$

$$A_{UT}^{\sin(3\phi_h - \phi_S)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

BM \otimes CF
Sivers \otimes FF
Transv \otimes CF
Pretz \otimes CF

Polarized Drell-Yan



DY

$$\frac{d\sigma^{LO}}{d\Omega} = \frac{\alpha_{em}^2}{Fq^2} F_U^1 \left\{ 1 + \cos^2 \theta + \sin^2 \theta \cos 2\varphi_{CS} A_U^{\cos 2\varphi_{CS}} \right. \\ \left. + S_T \begin{bmatrix} (1 + \cos^2 \theta) \sin \varphi_S A_T^{\sin \varphi_S} \\ + \sin^2 \theta \left(\begin{matrix} \sin(2\varphi_{CS} + \varphi_S) A_T^{\sin(2\varphi_{CS} + \varphi_S)} \\ + \sin(2\varphi_{CS} - \varphi_S) A_T^{\sin(2\varphi_{CS} - \varphi_S)} \end{matrix} \right) \end{bmatrix} \right\}$$

beam target

PDF \otimes PDF

BM \otimes BM
 f_1 \otimes Sivers
BM \otimes Transv
BM \otimes Pretz

$$A_T^{\cos 2\varphi_{CS}} \propto h_1^{\perp q} \otimes h_1^{\perp q}$$

$$A_T^{\sin \varphi_S} \propto f_1^q \otimes f_{1T}^{\perp q}$$

$$A_T^{\sin(2\varphi_{CS} - \varphi_S)} \propto h_1^{\perp q} \otimes h_{1T}^{\perp q}$$

$$A_T^{\sin(2\varphi_{CS} + \varphi_S)} \propto h_1^{\perp q} \otimes h_1^q$$

$$h_1^{\perp q} \Big|_{SIDIS} = -h_1^{\perp q} \Big|_{DY}$$

$$f_{1T}^{\perp q} \Big|_{SIDIS} = -f_{1T}^{\perp q} \Big|_{DY}$$

$$h_1^q \Big|_{SIDIS} = h_1^q \Big|_{DY}$$

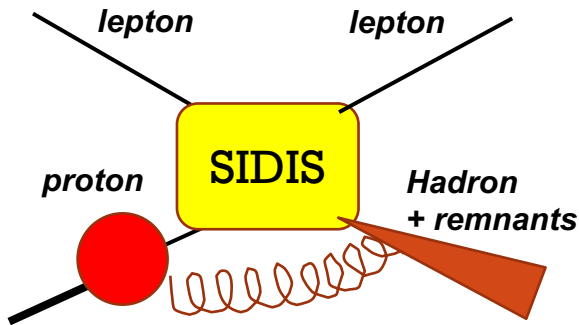
$$h_{1T}^{\perp q} \Big|_{SIDIS} = h_{1T}^{\perp q} \Big|_{DY}$$

* For these two processes TMD factorization is proven

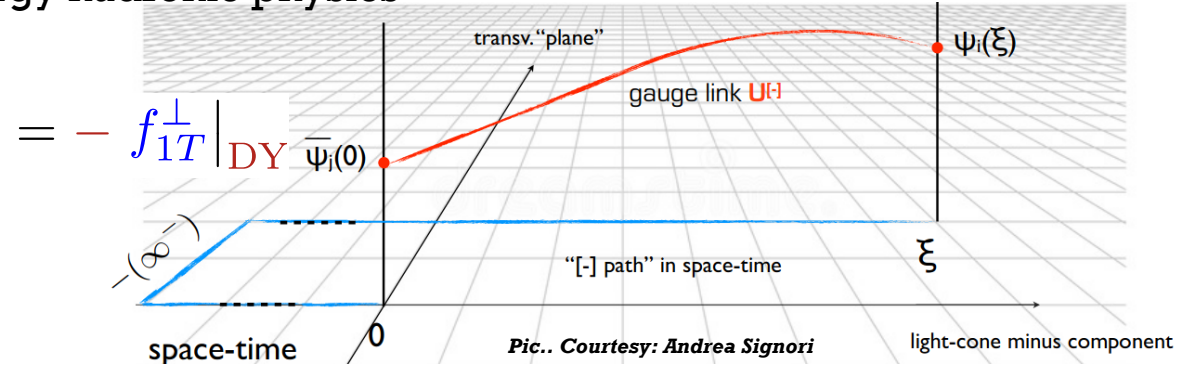
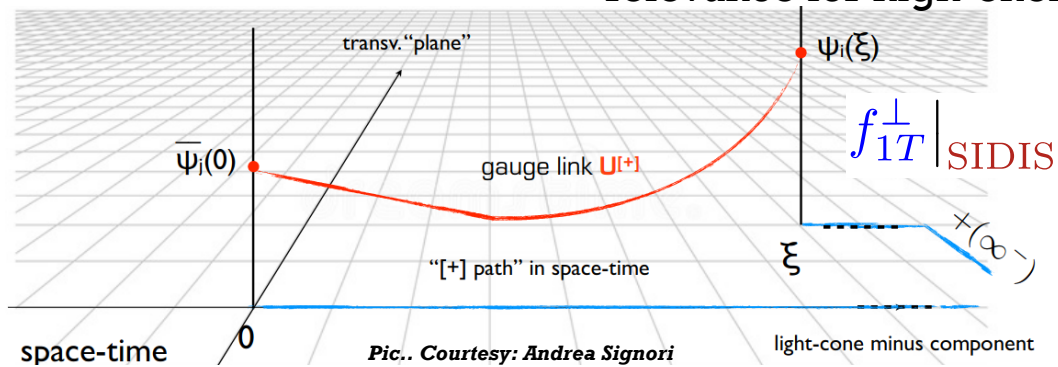
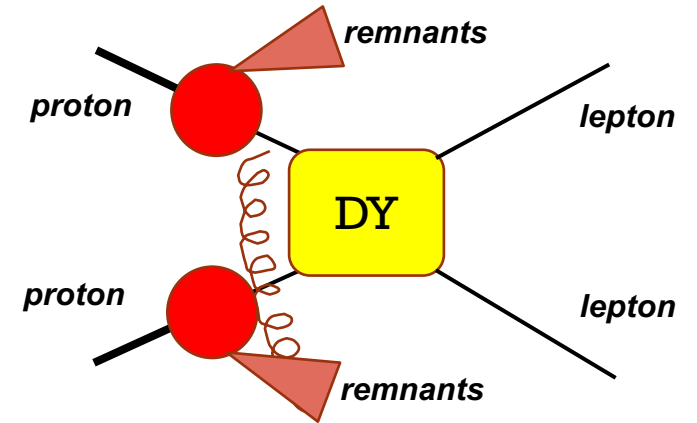
SIVERS FUNCTION

$$f_{q/p^\uparrow}(x, \mathbf{k}_T) = f_{q/p}(x, \mathbf{k}_T) + f_{1T}^\perp(x, \mathbf{k}_T) \mathbf{S} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_T)$$

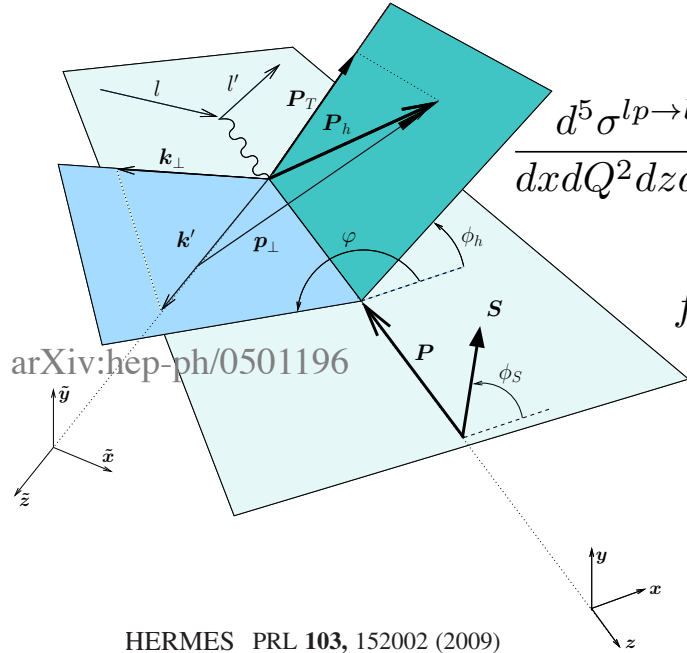
The Sivers function describes the correlation between the momentum direction of the struck quark and the spin of its parent nucleon.



- The gauge-invariant definition of the Sivers function predicts the opposite sign for the Sivers function in SIDIS compared to processes with color charges in the initial state and a colorless final state in Drell-Yan, $J/\psi, W^\pm, Z$
- This inclusion of the gauge link has profound consequences on factorization proofs and on the concept of universality, which are of fundamental relevance for high-energy hadronic physics

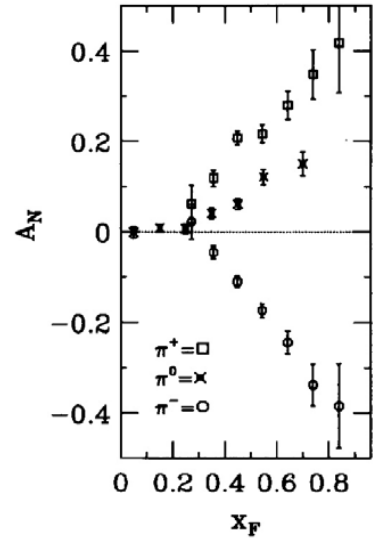


SIVERS ASYMMETRY FROM SIDIS



$$\frac{d^5 \sigma^{lp \rightarrow lhX}}{dx dQ^2 dz d^2 p_{hT}} \propto \sum_q e_q^2 \int d^2 \mathbf{k}_\perp \mathcal{K}(x, p_{hT}, Q^2) f_q(x, \mathbf{k}_\perp) D_{h/q}(z, p_\perp) + \mathcal{O}(\mathbf{k}_\perp/Q)$$

$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, k_\perp) - \frac{k_\perp}{m_p} f_{1T}^\perp(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \end{aligned}$$



Asymmetry in $pp^+ \rightarrow \pi X$ pion production from E704

Single Spin Asymmetry (Sivers Asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_s)}(x, y, z, p_{hT}) = \frac{d\sigma^{l\uparrow p \rightarrow hlX} - d\sigma^{l\downarrow p \rightarrow hlX}}{d\sigma^{l\uparrow p \rightarrow hlX} + d\sigma^{l\downarrow p \rightarrow hlX}} \equiv \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}$$

$$\begin{aligned} A_{UT}^{\sin(\phi_h - \phi_s)}(x, y, z, p_{hT}) &= \frac{[z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle] \langle k_S^2 \rangle^2}{[z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle] \langle k_\perp^2 \rangle^2} \exp \left[-\frac{p_{hT}^2 z^2 (\langle k_S^2 \rangle - \langle k_\perp^2 \rangle)}{(z^2 \langle k_S^2 \rangle + \langle p_\perp^2 \rangle) (z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle)} \right] \\ &\times \frac{\sqrt{2e} z p_{hT} \sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{M_1 \sum_q e_q^2 f_q(x) D_{h/q}(z)} \end{aligned}$$

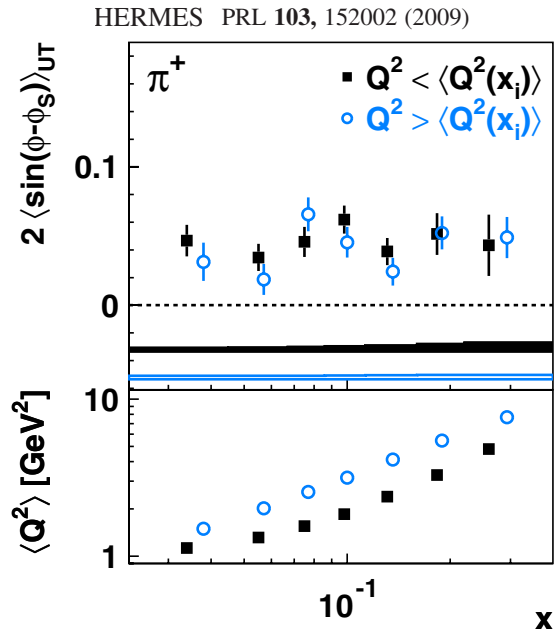
$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$A_{UT}^{\sin(\phi_h - \phi_s)}(x, y, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, M_1) \left(\frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$



SIVERS ASYMMETRY FROM DRELL-YAN

$$f_{q/p}(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle} \quad \langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$$

$$\begin{aligned} \Delta^N f_{q/p^{\uparrow}}(x, k_{\perp}) &= 2 \mathcal{N}_q(x) h(k_{\perp}) f_{q/p}(x, k_{\perp}) \\ &\equiv \Delta^N f_{q/p^{\uparrow}}(x) h(k_{\perp}) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle} \end{aligned}$$

$$\mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

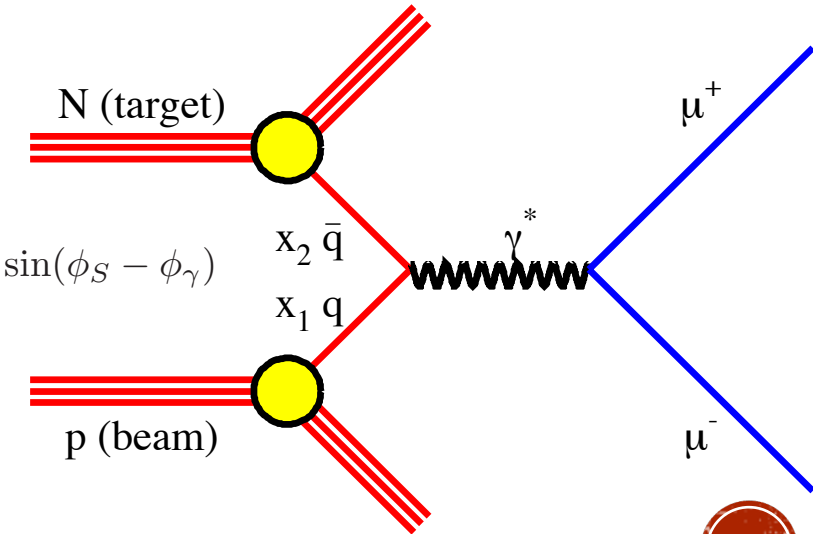
$$h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_1} e^{-k_{\perp}^2 / M_1^2}.$$

$$\frac{1}{\langle k_S^2 \rangle} = \frac{1}{M_1^2} + \frac{1}{\langle k_{\perp 1}^2 \rangle}$$

$$A_N^{\sin(\phi_{\gamma} - \phi_S)}(x_F, M, q_T) = \frac{\int d\phi_{\gamma} [N(x_F, M, q_T, \phi_{\gamma})] \sin(\phi_{\gamma} - \phi_S)}{\int d\phi_{\gamma} [D(x_F, M, q_T)]}$$

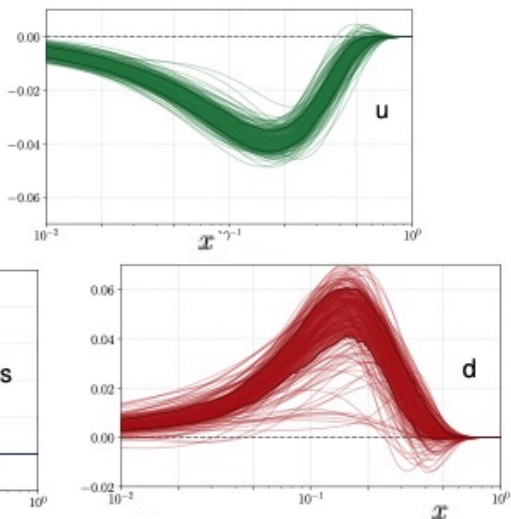
$$\begin{aligned} N(x_F, M, q_T, \phi_{\gamma}) &\equiv \frac{d^4\sigma^{\uparrow}}{dx_F dM^2 d^2\mathbf{q}_T} - \frac{d^4\sigma^{\downarrow}}{dx_F dM^2 d^2\mathbf{q}_T} \\ &= \frac{4\pi\alpha^2}{9M^2s} \sum_q \frac{e_q^2}{x_1 + x_2} \Delta^N f_{q/A^{\uparrow}}(x_1) f_{\bar{q}/B}(x_2) \sqrt{2e} \frac{q_T}{M_1} \frac{\langle k_S^2 \rangle^2 \exp[-q_T^2 / (\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle)]}{\pi [\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle]^2 \langle k_{\perp 2}^2 \rangle} \sin(\phi_S - \phi_{\gamma}) \end{aligned}$$

$$\begin{aligned} D(x_F, M, q_T) &\equiv \frac{1}{2} \left[\frac{d^4\sigma^{\uparrow}}{dx_F dM^2 d^2\mathbf{q}_T} + \frac{d^4\sigma^{\downarrow}}{dx_F dM^2 d^2\mathbf{q}_T} \right] = \frac{d^4\sigma^{unp}}{dx_F dM^2 d^2\mathbf{q}_T} \\ &= \frac{4\pi\alpha^2}{9M^2s} \sum_q \frac{e_q^2}{x_1 + x_2} f_{q/A}(x_1) f_{\bar{q}/B}(x_2) \frac{\exp[-q_T^2 / (\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle)]}{\pi [\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle]} \end{aligned}$$



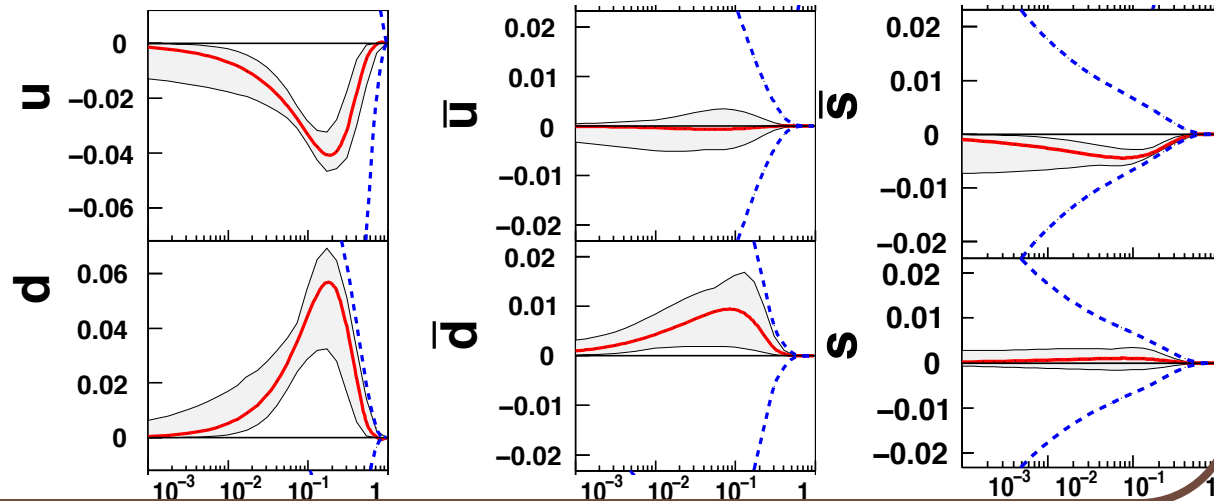
GLOBAL ANALYSES OF SIVERS FUNCTION

A. Bacchetta, F. Delcarro, C. Pasiano, M. Radici
arXiv 2004.14278 (2020)

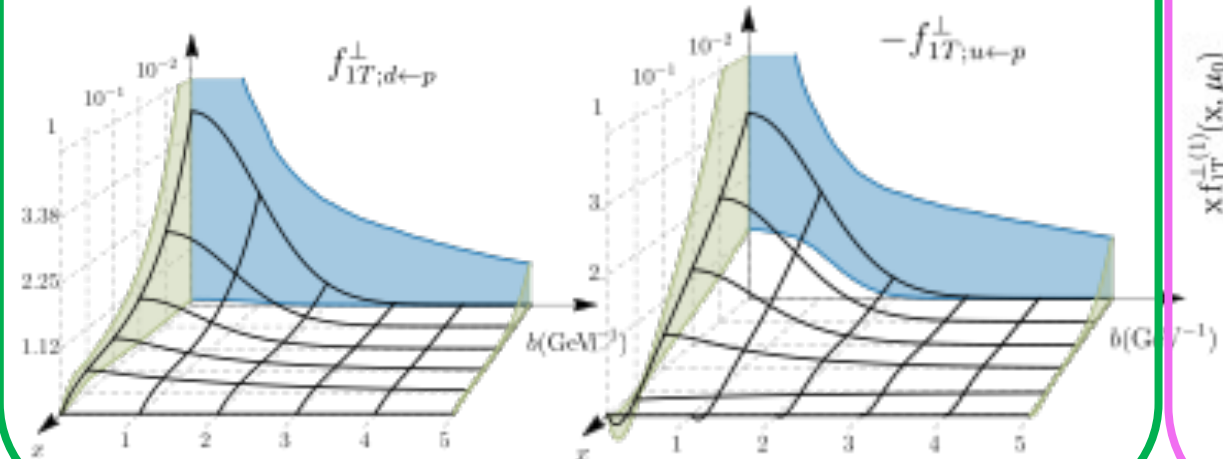


HERMES (2020)
COMPASS (2009)
COMPASS (2015)
JLab (2011)

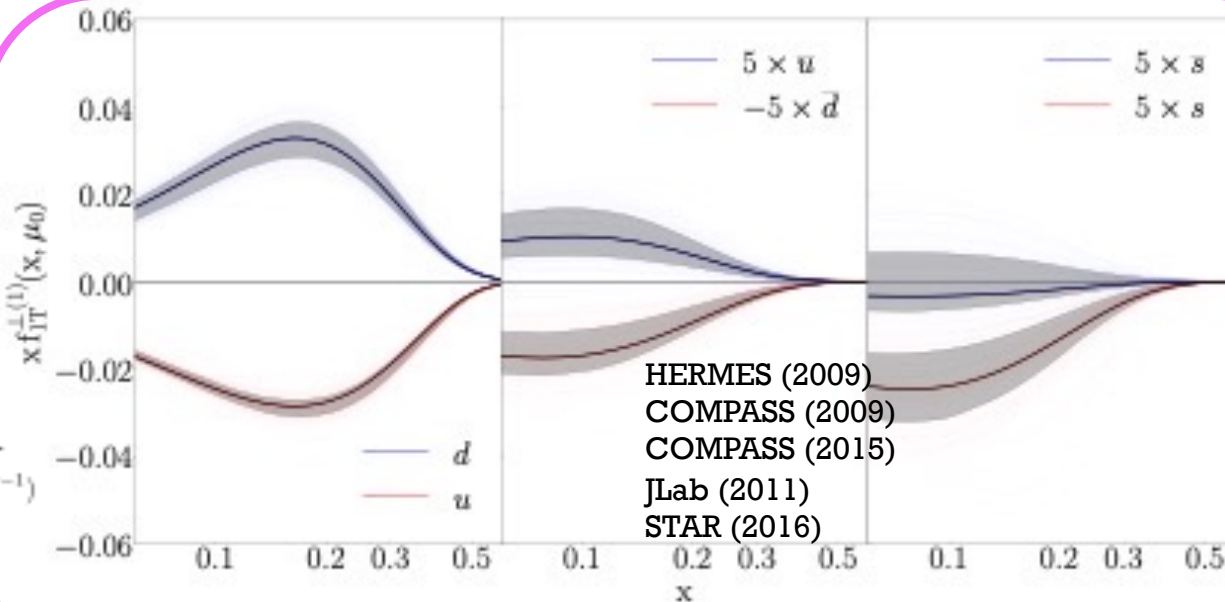
M. Anselmino, M. Boglion, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin_PRD_79_54010_(2009)



HERMES (2020), COMPASS (2009), COMPASS (2015)
JLab (2011), STAR (2016), COMPASS DY (2017)



M. Bury, A. Prokudin, A. Vladimirov, JHEP_05_151 (2021)



HERMES (2009)
COMPASS (2009)
COMPASS (2015)
JLab (2011)
STAR (2016)

M. Echevarria, Z. Kang, J. Terry_JHEP_01_126_(2021)

FITTING METHODOLOGY

Inputs:

- Unpolarized PDFs : LHAPDF6 (CTEQ61)
- Fragmentation Functions:
 - Pi+: NNFF10_Pip_nlo
 - Pi- : NNFF10_Pim_nlo
 - Pi0: NNFF10_Pisum_nlo
 - K+: NNFF10_Kap_nlo
 - K- : NNFF10_Kam_nlo

V. Bertone et. al arXiv:1706.07049

Fit parameters (13):

$$M_1$$

$$N_u, \alpha_u, \beta_u, N_{\bar{u}}$$

$$N_d, \alpha_d, \beta_d, N_{\bar{d}}$$

$$N_s, \alpha_s, \beta_s, N_{\bar{s}}$$

Data Sets (on consideration):

SIDIS

- HERMES_p_2009 (from Luciano Pappalardo)
- COMPASS_d_2009 (from Bakur Parsamyan)
- COMPASS_p_2015 (from Bakur Parsamyan)
- HERMES_p_2020 (from Luciano Pappalardo)

DY

- COMPASS_2017 (from Bakur Parsamyan)

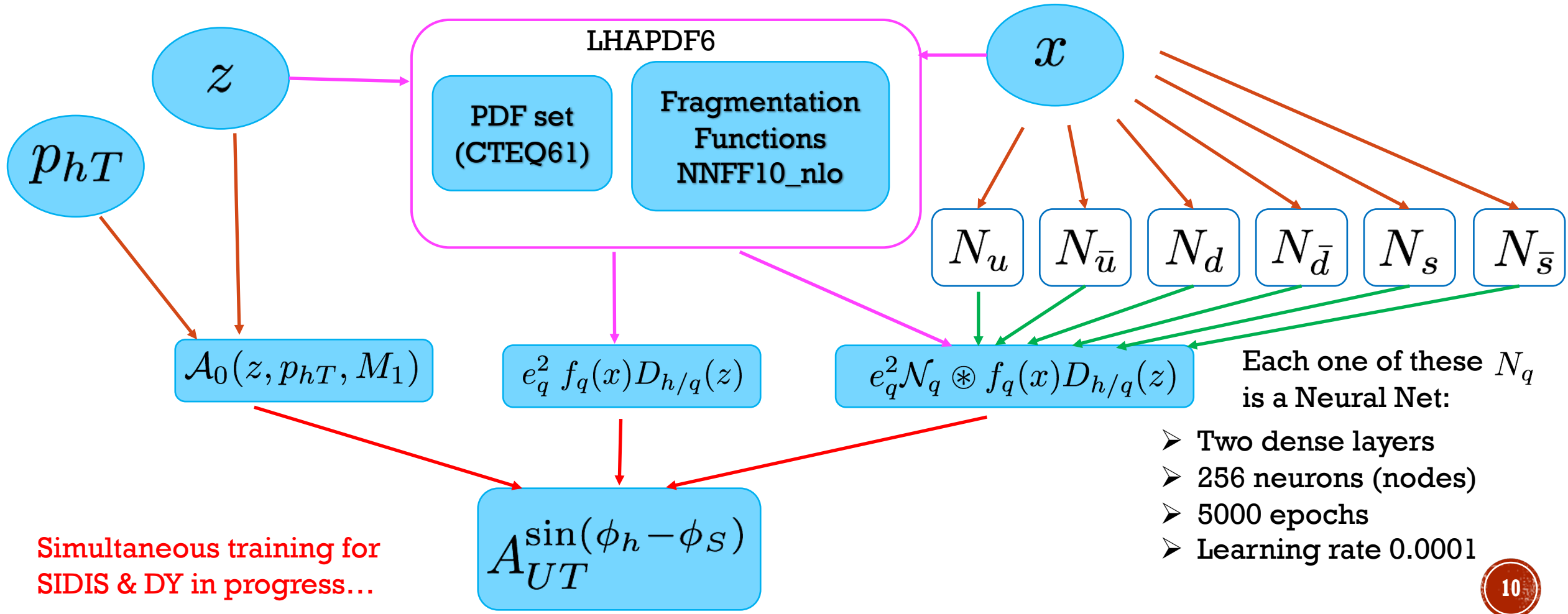
Fitting routines:

- “iminuit” (python supported version of MINUIT)
- Using a Neural Network approach

NEURAL NETWORK APPROACH WITH SIDIS DATA

Motivation for ANN

$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, y, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, M_1) \left(\frac{\sum_q \mathcal{N}_q e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$



Simultaneous training for SIDIS & DY in progress...

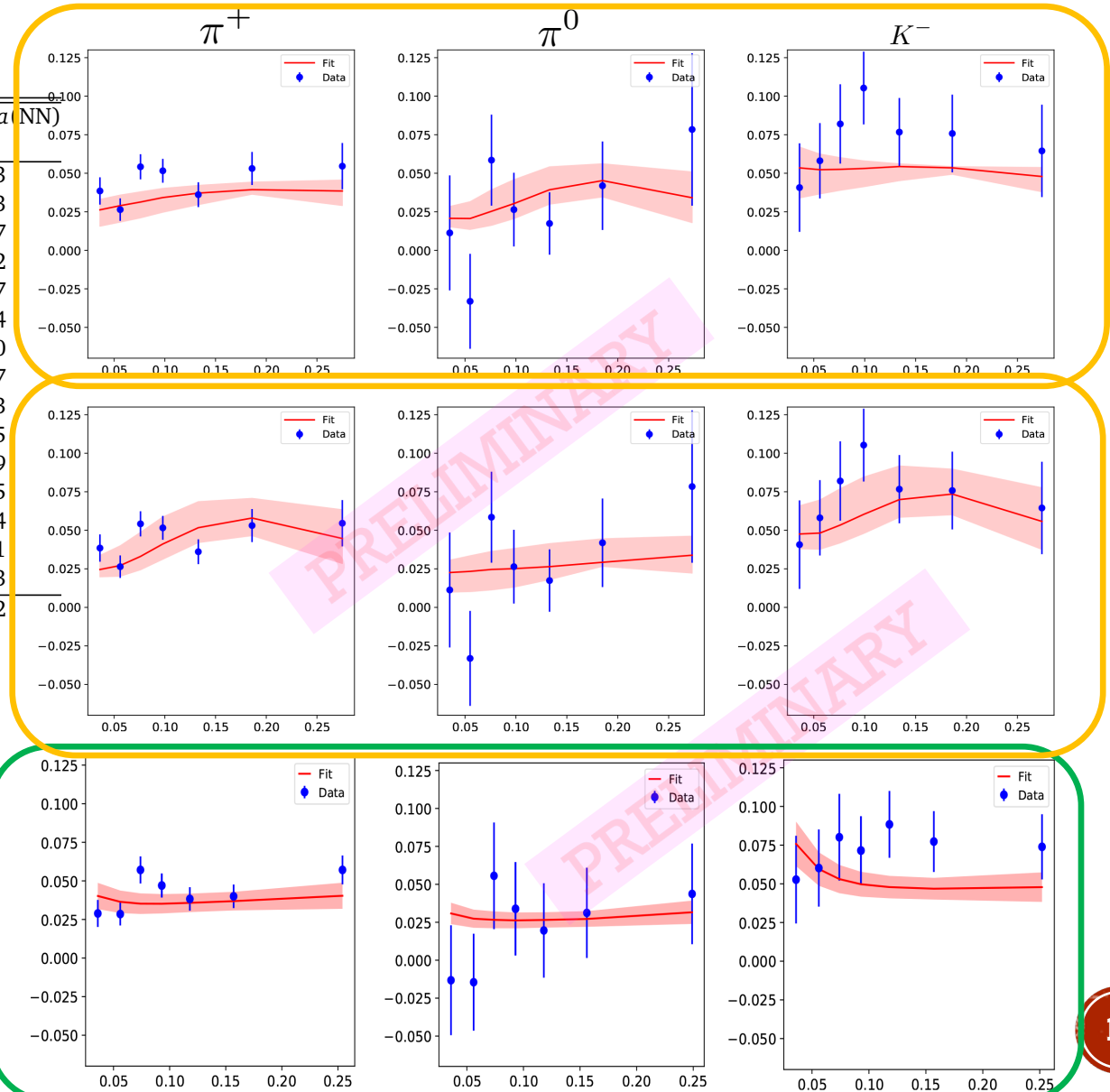
FITS TO SIDIS DATA : INITIAL ATTEMPTS

Individual fits

Hadron	Dependence	HERMES2009		HERMES2020			
		ndata	$\chi^2/ndata$	$\chi^2/ndata(NN)$	ndata	$\chi^2/ndata$	$\chi^2/ndata(NN)$
π^+	x	7	2.53	2.29	8	2.12	2.23
π^+	z	7	1.02	1.01	11	1.49	1.63
π^+	P_{hT}	7	5.23	3.40	8	1.14	2.07
π^-	x	7	1.94	3.13	8	1.81	2.82
π^-	z	7	2.45	0.52	11	1.16	0.57
π^-	P_{hT}	7	1.61	1.96	8	1.20	1.44
π^0	x	7	0.85	0.90	8	0.40	0.50
π^0	z	7	1.11	1.13	11	0.95	0.97
π^0	P_{hT}	7	2.00	1.61	8	0.50	0.73
K^+	x	7	1.22	1.78	8	0.48	1.45
K^+	z	7	2.97	3.69	11	6.31	7.99
K^+	P_{hT}	7	2.65	1.29	8	1.26	2.45
K^-	x	7	0.49	0.52	8	0.26	0.54
K^-	z	7	0.52	0.57	10	0.93	1.11
K^-	P_{hT}	7	0.96	0.73	8	0.79	2.93
Total		105	1.84	1.64	134	1.477	2.02

Parameter	HERMES 2009	HERMES2020
M_1	1.303 ± 0.010	7.590 ± 0.008
N_u	0.169 ± 0.002	0.960 ± 0.084
α_u	0.645 ± 0.125	2.291 ± 0.200
β_u	3.122 ± 2.661	9.826 ± 1.556
$N_{\bar{u}}$	0.007 ± 0.003	0.205 ± 0.02
N_d	-0.434 ± 0.005	-4.713 ± 0.004
α_d	1.777 ± 0.909	0.482 ± 0.866
β_d	7.788 ± 2.144	$(5.675 \pm 6.45) \times 10^{-6}$
$N_{\bar{d}}$	-0.142 ± 0.048	1.490 ± 0.05
N_s	0.563 ± 0.073	4.528 ± 0.073
α_s	$(6.84 \pm 10.00) \times 10^{-5}$	$(1.745 \pm 9.20) \times 10^{-5}$
β_s	$(5.987 \pm 8.77) \times 10^{-10}$	$(6.082 \pm 9.55) \times 10^{-10}$
$N_{\bar{s}}$	-0.122 ± 0.504	8.692 ± 0.46

Projected
Asymmetries
For HERMES 2020
Trained based on
HERMES 2009



HERMES 2009

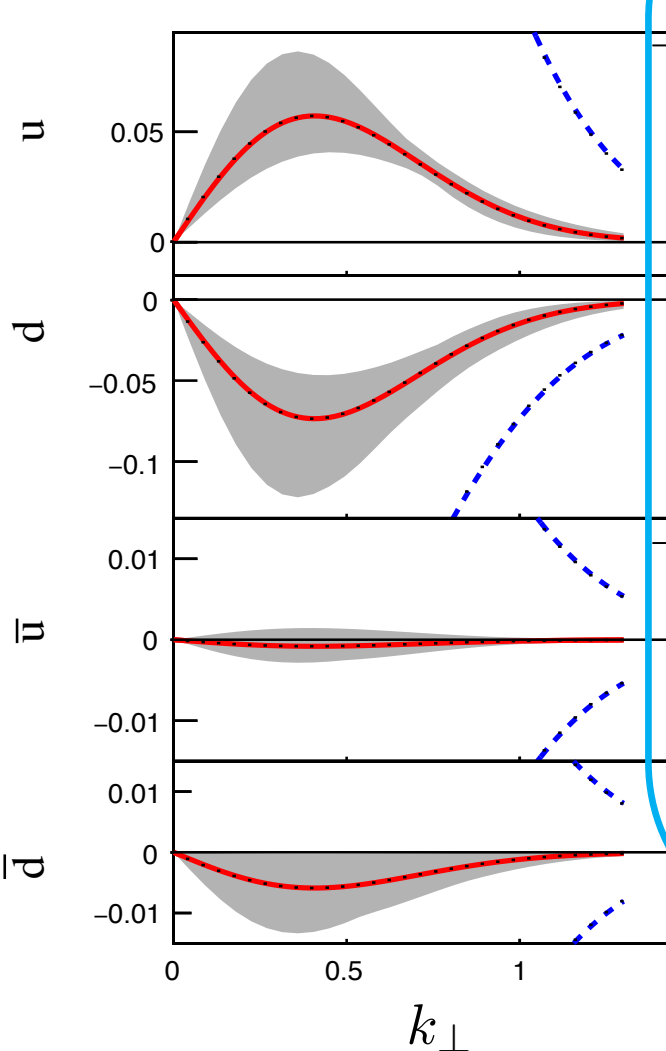
HERMES 2020

GLOBAL FIT TO SIDIS DATA

HERMES2009, COMPASS2009,
COMPASS2015

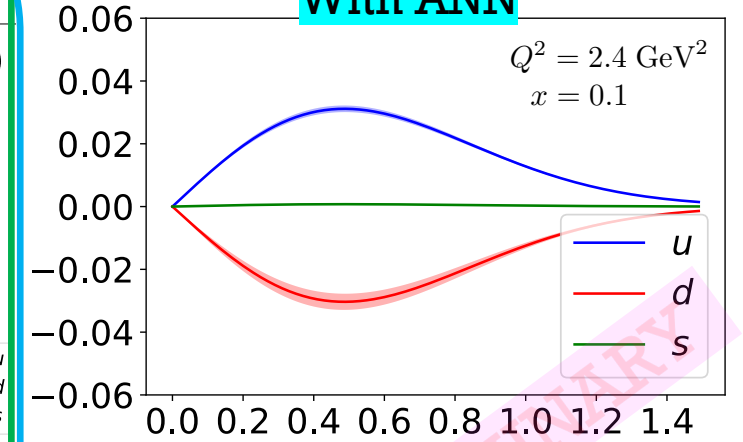
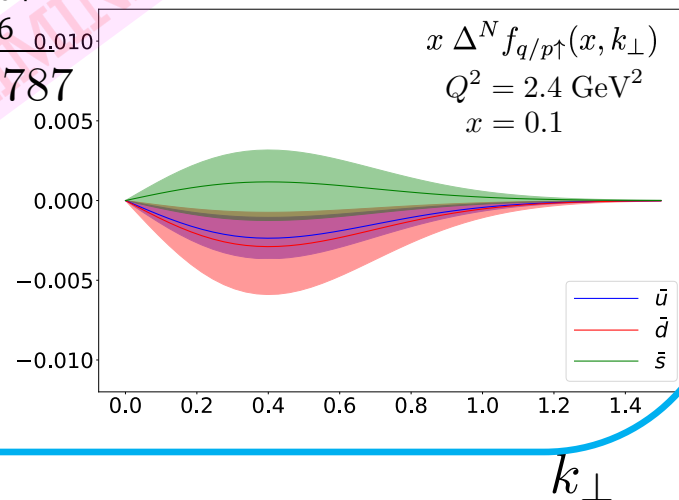
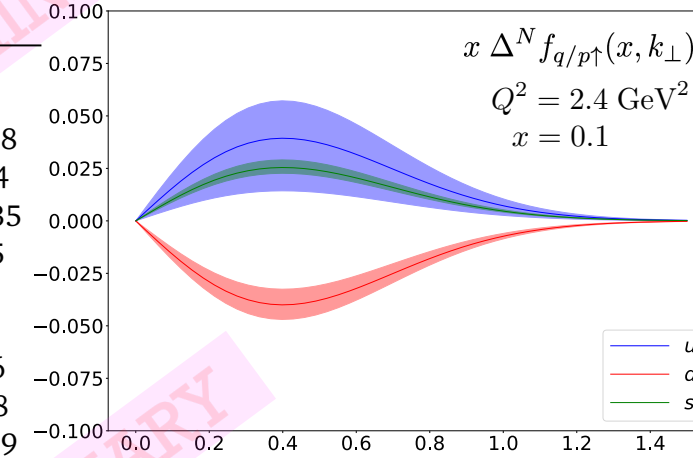
HERMES2009, **HERMES2020**
COMPASS2009, COMPASS2015

HERMES2009, **HERMES2020**
COMPASS2009, COMPASS2015
With ANN

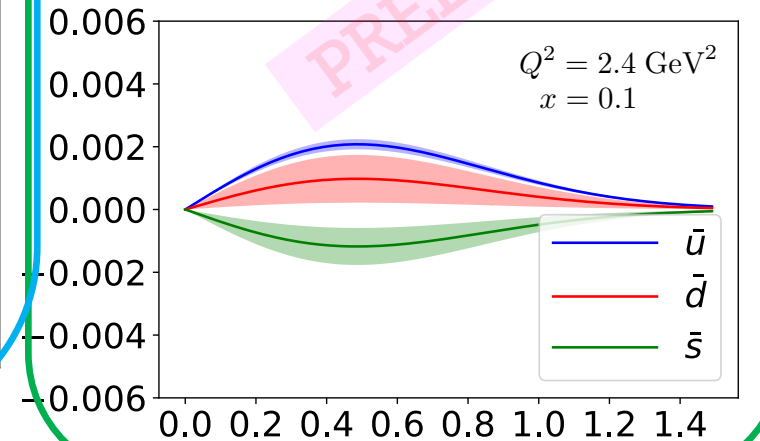


Parameter	Value
M_1	5.35 ± 0.16
N_u	0.62 ± 0.12
α_u	2.537 ± 0.018
β_u	13.82 ± 0.14
$N_{\bar{u}}$	-0.189 ± 0.035
N_d	-1.15 ± 0.05
α_d	2.2 ± 0.05
β_d	9.12 ± 0.33
$N_{\bar{d}}$	-0.16 ± 0.06
N_s	10.09 ± 0.18
α_s	0.497 ± 0.019
β_s	0.066 ± 0.004
$N_{\bar{s}}$	0.18 ± 0.16

$\chi^2/\text{NDF} = 1.787$



NN Trained 90% of SIDIS Loss ~ 1.82



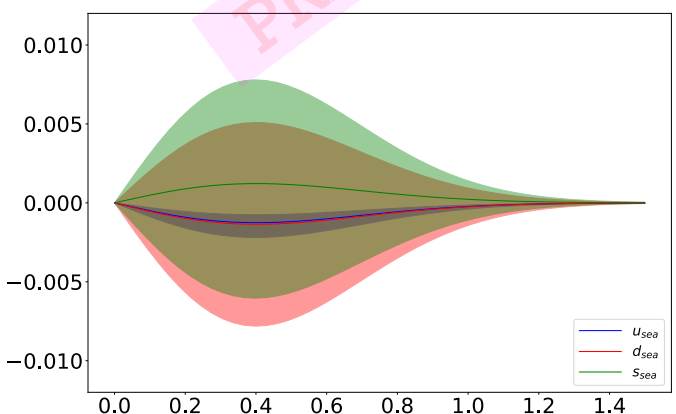
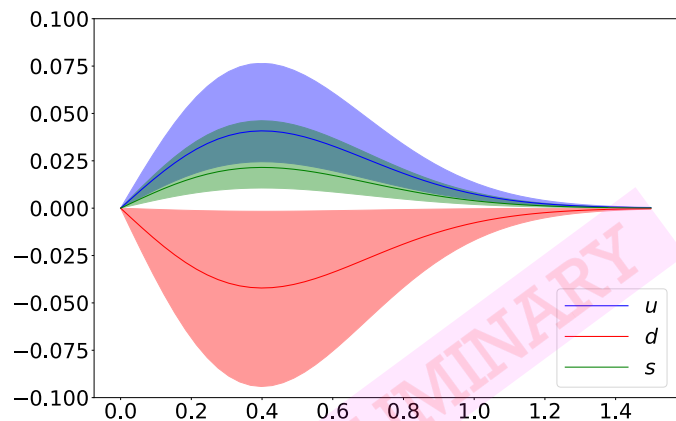
ANN: with few replicas \rightarrow need to generate with higher number of replicas

PRELIMINARY

GLOBAL FIT TO SIDIS & DY DATA

PRELIMINARY

With sign change

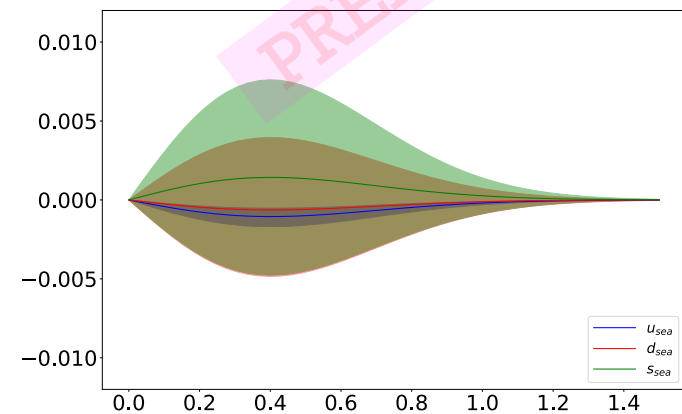
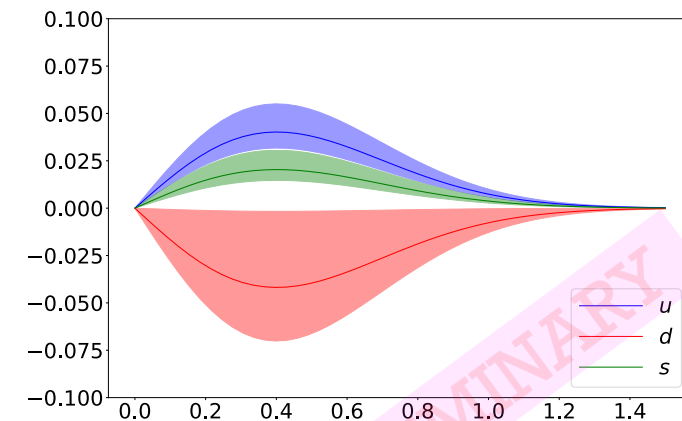


Parameter	sign-flip	no-sign-flip
M_1	5.7 ± 0.8	6.1 ± 0.5
N_u	0.69 ± 0.08	0.72 ± 0.05
α_u	2.74 ± 0.09	2.71 ± 0.05
β_u	15.1 ± 0.6	15.05 ± 0.30
$N_{\bar{u}}$	-0.107 ± 0.017	-0.096 ± 0.018
N_d	-1.34 ± 0.15	-1.30 ± 0.11
α_d	1.6 ± 0.4	1.36 ± 0.31
β_d	5.4 ± 2.5	4.7 ± 1.8
$N_{\bar{d}}$	-0.08 ± 0.13	-0.04 ± 0.12
N_s	11.2 ± 1.4	12.0 ± 0.9
α_s	0.85 ± 0.09	0.91 ± 0.05
β_s	0.46 ± 0.12	0.52 ± 0.07
$N_{\bar{s}}$	0.2 ± 0.4	0.25 ± 0.32
χ^2/N	1.871	1.870

Ongoing work:

- Analyzing the fit results & optimizing the fitting framework
- DY extension to the SIDIS NN model

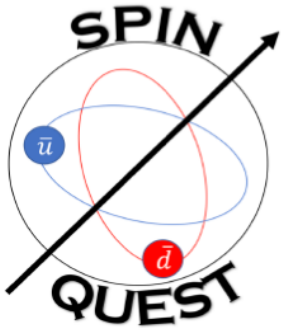
Without sign change



k_{\perp}

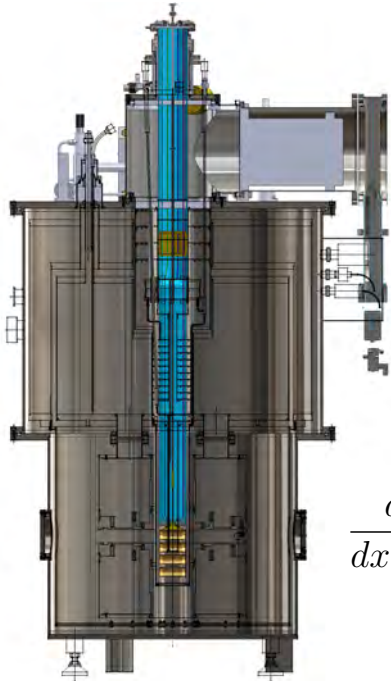
DISCUSSION & FUTURE WORK

- Performing simultaneous fits to SIDIS and DY data with higher statistics of replicas (on-going).
- Improving the Neural Network to train simultaneously on both SIDIS & DY data with optimizing hyperparameters with higher statistics of replicas.
- Investigating towards Sivers Asymmetry extraction from Drell –Yan with/without considering the “sign-flip” of the Sivers Function.
- Simultaneous fits to Sivers function and Boer-Mulders function (on-going).



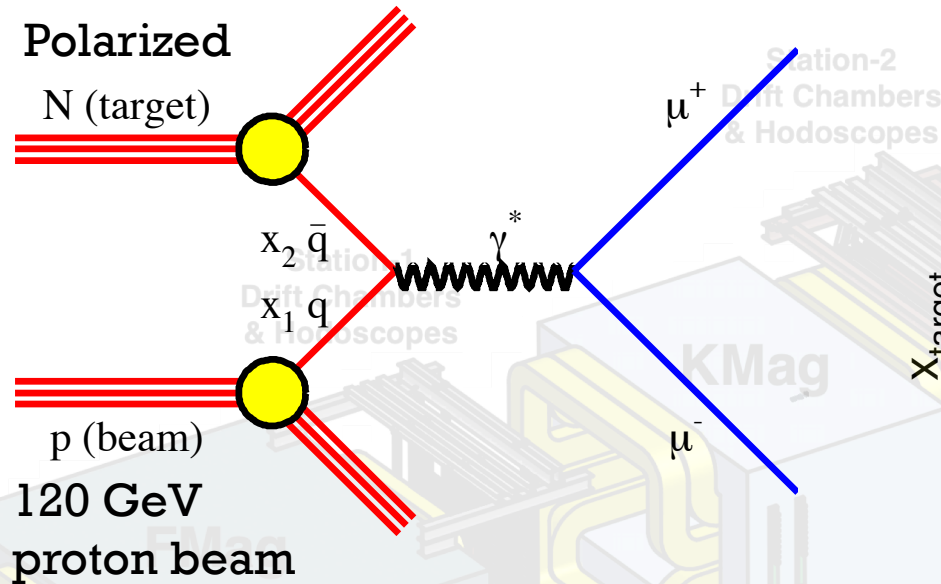
SPINQUEST (E1039) EXPERIMENT AT FERMILAB

➤ Measurement of 'sea' quark Sivers function



LANL-UVA
Polarized Target

$$pp \uparrow (d^\uparrow) \rightarrow \mu^+ \mu^- X, 4 < M_{\mu\mu} < 9 \text{ GeV}$$

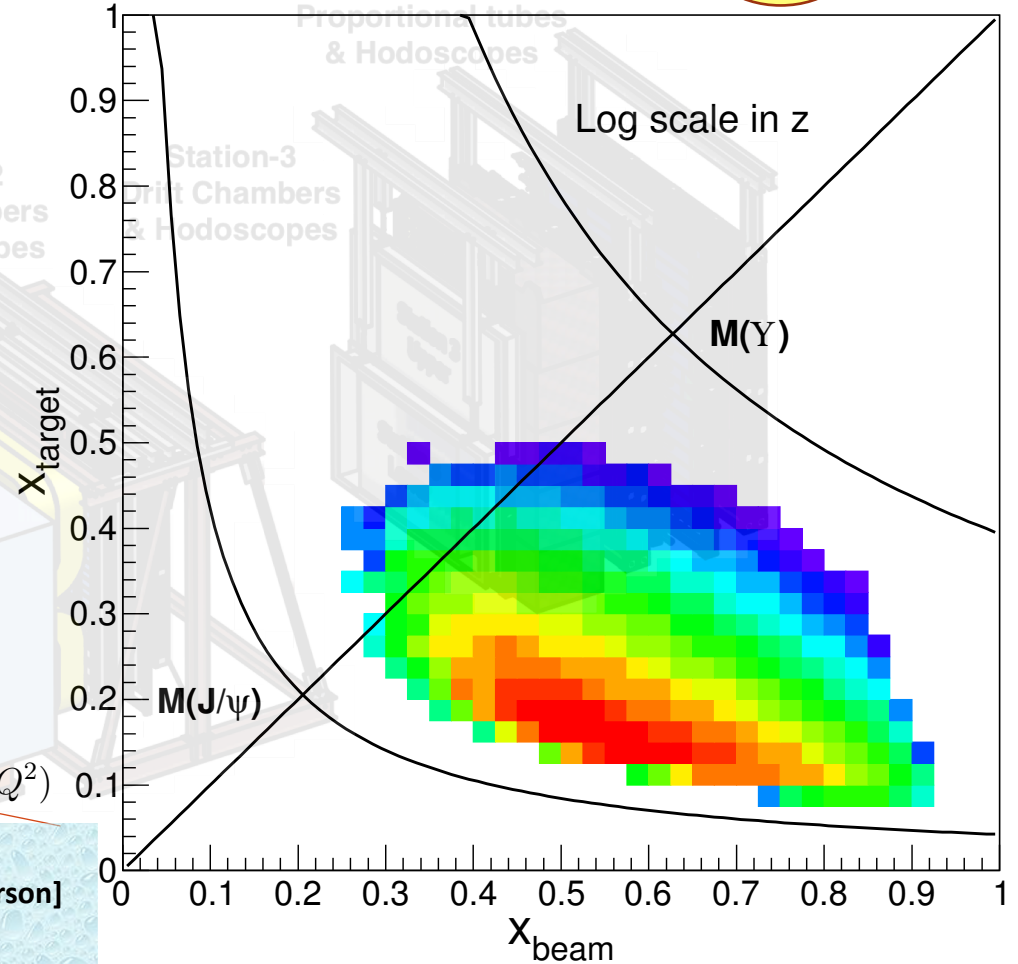


$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_i e_i^2 (q_i^B(x_1, Q^2) \bar{q}_i^T(x_2, Q^2) + \bar{q}_i^B(x_1, Q^2) q_i^T(x_2, Q^2))$$

Please Join The Effort
Dustin Keller (dustin@virginia.edu)[Spokesperson]
Kun Liu ([Spokesperson])

<https://spinquest.fnal.gov/>

<http://twist.phys.virginia.edu/E1039/>



Thank you



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